The Lattice Fokker-Planck Method

Santosh Ansumali[1]

[1] *Engineering Mechanics Unit, Jawaharlal Nehru Centre for Advanced Scientific Research, Jakkur, Bangalore 560064, India*

A probabilistic description of observables in terms of Fokker-Planck equation is one of the important modeling tool in the theoretical description of dynamics encountered in non-equilibrium statistical systems. Typical approach to solve such description is to work with equivalent Langevin equation and use Brownian dynamics to solve the resulting stochastic differential equation. In the context of complex fluids, very often one is interested in solving a coupled system consisting of hydrodynamics description in terms of Navier-Stokes equation and Brownian dynamics solver for appropriate Fokker-Planck equation[1]. Typically, resulting system is computationally too intensive for wide variety of practical purposes. Thus, for such computationally demanding situation, it is typical to resort to reduced description in terms of few relevant moments of the distribution function [2].

Though, it is widely recognized that a deterministic solver of low dimensional Fokker-Planck will be always much faster in comparison to the Brownian dynamics, they are rarely used due to the fact that no efficient and well accepted methodology exist for direct solution of the Fokker-Planck equation [3]. Moreover, in the case of highly non-equilibrium situations lower order truncation of eigenmodes of Fokker-Planck does not predict dynamics correctly [4, 5]. The aim of the present discussion is to describe an efficient algorithm to discretize Fokker-Planck equation. The distinct existence of slower manifold for probability density associated with internal degrees of freedom suggests that the situation is apt for bottom-up modeling approach via lattice-Boltzmann model. It can further be shown that the actual diffusive dynamics, governing the momentum relaxation described by a Fokker-Planck equation, may be replaced by a BGK-type relaxation dynamics without affecting the slow dynamics in the space governing internal degrees of freedom [6–8].

REFERENCES

[1] H. C. Öttinger, B. Van Den Brule, and M. Hulsen, Journal of non-newtonian fluid mechanics **70**, 255 (1997).
[2] R. Sureshkumar, A. Beris, and R. Handler, Physics of Fluids **9**, 743 (1997).
[3] A. Lozinski and C. Chauvière, Journal of Computational Physics **189**, 607 (2003).
[4] D. Moroni, B. Rotenberg, J. Hansen, S. Succi, and S. Melchionna, Physical Review E **73**, 66707 (2006).
[5] H. Risken, *The Fokker-Planck equation: Methods of solution and applications* (Springer Verlag, 1996).
[6] S. Singh, G. Subramanian, and S. Ansumali, Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences **369**, 2301 (2011).
[7] A. Ammar, Journal of Non-Newtonian Fluid Mechanics **165**, 1082 (2010).
[8] S. Singh, G. Subramanian, and S. Ansumali, Under Review (2013).