Pattern recognition by covariogram

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Let D be a convex body in n-dimensional Euclidean space R^n, that is a compact, convex subset of R^n, with non-empty interior. The n-dimensional Lebesgue measure in R^n is denoted by $L_n(\cdot)$. If $h \in R^n$, then $D + h$ denotes the translate of D by h. The covariogram of a convex body $D \subset R^n$ is the following function $C(D, h) = L_n(D \cap (D + h))$. G. Matheron conjectured that a planar convex body is uniquely determined by its covariogram, up to translation and reflection. Bianchi (see [2]) found counterexamples to the covariogram conjecture in dimensions greater than or equal to 4, and a positive answer for three-dimensional polytopes. The general three-dimensional case is still open. Denote by $F_D(u, x)$ orientation-dependent chord length distribution function. Determination of a convex body D by these distributions, for all directions, is equivalent to the determination by its covariogram. Matheron (see [2]) obtained relationship between $F_D(u, x)$ and covariogram. The applications in both geometric and computer tomography are well known (see [1]). In the paper [3] is proved that for any finite subset A of directions, there are two non-congruent domains for which orientation-dependent chord length distribution functions coincide for any direction from A (see also [4]).

References

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